

Ghost Chaplygin scalar field model of dark energy

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Abstract

We investigate the correspondence between the ghost and Chaplygin gas dark energy models in the framework of Einstein gravity. We consider a spatially non-flat FRW universe containing the interacting dark energy (DE) with dark matter (DM). We reconstruct the potential and the dynamics for the Chaplygin gas scalar field model according to the evolutionary behavior of the ghost dark energy (GDE) which describes the phantom accelerated expansion of the universe.

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1 Interacting GDE with DM

Here, we investigate the GDE model in the framework of Einstein gravity. The GDE density is given by [1, 2]

$$\rho_{\Lambda} = \alpha H, \quad (1)$$

where α is a constant.

We consider a spatially non-flat FRW universe containing the interacting GDE with dark matter (DM). The first Friedmann equation in the standard FRW cosmology is

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}(\rho_m + \rho_{\Lambda}), \quad (2)$$

where $M_p = (8\pi G)^{-1/2}$ is the reduced Planck mass and $H = \dot{a}/a$ is the Hubble parameter. Here, $k = 0, 1, -1$ represent a flat, closed and open FRW universe, respectively. Also ρ_m and ρ_{Λ} are the energy densities of DM and GDE, respectively.

Using the usual definitions for the dimensionless energy densities as

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3M_p^2 H^2}, \quad \Omega_{\Lambda} = \frac{\rho_{\Lambda}}{\rho_{cr}} = \frac{\rho_{\Lambda}}{3M_p^2 H^2}, \quad \Omega_k = \frac{k}{a^2 H^2}, \quad (3)$$

the Friedmann equation (2) can be rewritten as

$$1 + \Omega_k = \Omega_m + \Omega_{\Lambda}. \quad (4)$$

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Substituting Eq. (1) into $\rho_\Lambda = 3M_p^2 H^2 \Omega_\Lambda$ yields

$$\Omega_\Lambda = \frac{\alpha}{3M_p^2 H}. \quad (5)$$

Using the above relation, the curvature energy density parameter can be obtained as

$$\Omega_k = \left(\frac{9M_p^4 k}{\alpha^2} \right) \frac{\Omega_\Lambda^2}{a^2} = \left(\frac{\Omega_{k0}}{\Omega_{\Lambda0}^2} \right) \frac{\Omega_\Lambda^2}{a^2}, \quad (6)$$

where the index 0 denotes the value of a quantity at present.

In the presence of interaction between GDE and pressureless DM ($p_m = 0$), the energy equations are given by

$$\dot{\rho}_\Lambda + 3H(1 + \omega_\Lambda)\rho_\Lambda = -Q, \quad (7)$$

$$\dot{\rho}_m + 3H\rho_m = Q, \quad (8)$$

where $\omega_\Lambda = p_\Lambda/\rho_\Lambda$ is the equation of state (EoS) parameter of the GDE. Also $Q = 3b^2 H \rho_\Lambda$ stands for the interaction term with the coupling constant b^2 .

Taking time derivative of Eq. (1) and using (2), (7) and (8) gives

$$\frac{\dot{\rho}_\Lambda}{\rho_\Lambda} = -\frac{H}{2}[\Omega_k + 3(1 + \omega_\Lambda\Omega_\Lambda)]. \quad (9)$$

With the help of Eqs. (7) and (9), the EoS parameter of the GDE can be obtained as

$$\omega_\Lambda = -\frac{1}{2 - \Omega_\Lambda} \left(1 - \frac{\Omega_k}{3} + 2b^2 \right). \quad (10)$$

Equation (10) shows that in the absence of interaction, i.e. $b^2 = 0$, at the late time where $\Omega_\Lambda \rightarrow 1$ and $\Omega_k \rightarrow 0$, we have $\omega_\Lambda = -1$ which behaves like Λ CDM. Besides, taking $\Omega_\Lambda = 0.73$ and $\Omega_k = 0.01$ for the present time then we obtain $\omega_\Lambda = -0.78$ which acts like the quintessence DE. But in the presence of interaction, taking again $\Omega_\Lambda = 0.73$ and $\Omega_k = 0.01$ for the present time then from Eq. (10) the EoS parameter can behave like phantom DE $\omega_\Lambda < -1$ provided $b^2 > 0.14$. This value for coupling constant b^2 is consistent with the observations [3]. Also the phantom divide crossing is compatible with the recent observations [4].

Taking time derivative of Eq. (5) and using (2), (7), (8) and (10) one can obtain the evolution of the GDE density parameter as

$$\frac{\Omega'_\Lambda}{\Omega_\Lambda} = \frac{3}{2} \left[1 + \frac{\Omega_k}{3} - \frac{\Omega_\Lambda}{2 - \Omega_\Lambda} \left(1 - \frac{\Omega_k}{3} + 2b^2 \right) \right], \quad (11)$$

where $\Omega'_\Lambda = \dot{\Omega}_\Lambda/H = d\Omega_\Lambda/d \ln a$.

2 Interacting Chaplygin gas scalar field with DM

The EoS of the Chaplygin gas model of DE is as follows [5]

$$p_{\text{Ch}} = -\frac{A}{\rho_{\text{Ch}}}, \quad (12)$$

where A is a positive constant. Inserting the above EoS into the energy equation (7) leads to a density evolving as

$$\rho_{\text{Ch}} = \frac{1}{\sqrt{1+b^2}} \left(A + \frac{B}{a^{6(1+b^2)}} \right)^{\frac{1}{2}}, \quad (13)$$

where B is a positive integration constant.

Using Eqs. (12) and (13) the EoS parameter of the Chaplygin gas DE is obtained as

$$\omega_{\text{Ch}} = \frac{p_{\text{Ch}}}{\rho_{\text{Ch}}} = -1 - \frac{Ab^2 - Ba^{-6(1+b^2)}}{A + Ba^{-6(1+b^2)}}, \quad (14)$$

which shows that for $Ab^2 > Ba^{-6(1+b^2)}$ we have $\omega_{\text{Ch}} < -1$ which corresponds to a universe dominated by phantom DE. In other words, crossing the phantom divide line occurs when

$$a > a_{\min} = \left(\frac{B}{Ab^2} \right)^{\frac{1}{6(1+b^2)}}, \quad (15)$$

which corresponds to a bouncing universe [6]. Note that in the absence of interaction ($b^2 = 0$), Eq. (14) gives $\omega_{\text{Ch}} > -1$ which corresponds to a universe dominated by quintessence DE.

Here, one can obtain a corresponding potential for the Chaplygin gas DE by treating it as an ordinary scalar field ϕ . Using Eqs. (12) and (13) together with $\rho_{\text{Ch}} = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ and $p_{\text{Ch}} = \frac{1}{2}\dot{\phi}^2 - V(\phi)$, we find

$$\dot{\phi}^2 = -\frac{Ab^2 - Ba^{-6(1+b^2)}}{(1+b^2)^{\frac{1}{2}}[A + Ba^{-6(1+b^2)}]^{\frac{1}{2}}}, \quad (16)$$

$$V(\phi) = \frac{2(1+b^2)A + Ba^{-6(1+b^2)}}{2(1+b^2)^{\frac{1}{2}}[A + Ba^{-6(1+b^2)}]^{\frac{1}{2}}}. \quad (17)$$

Equation (16) clears that for $Ab^2 > Ba^{-6(1+b^2)}$, the Chaplygin gas DE behaves like a phantom scalar field, i.e. $\dot{\phi}^2 < 0$, whereas in the absence of interaction it acts like a quintessence scalar field ($\dot{\phi}^2 > 0$).

3 Correspondence between GDE and Chaplygin gas

Here, we establish a correspondence between the GDE and Chaplygin gas scalar field model. To do this, equating Eqs. (1) and (13), i.e. $\rho_{\Lambda} = \rho_{\text{Ch}}$, gives

$$A = \alpha^2 H^2 (1 + b^2) - Ba^{-6(1+b^2)}. \quad (18)$$

Also equating Eqs. (10) and (14), i.e. $\omega_{\Lambda} = \omega_{\text{Ch}}$, and using (18) we obtain

$$A = \frac{\alpha^2 H^2}{2 - \Omega_{\Lambda}} \left(1 - \frac{\Omega_k}{3} + 2b^2 \right). \quad (19)$$

Substituting Eq. (19) into (18) yields

$$B = \alpha^2 H^2 a^{6(1+b^2)} \left[(1 + b^2) - \frac{1}{2 - \Omega_{\Lambda}} \left(1 - \frac{\Omega_k}{3} + 2b^2 \right) \right]. \quad (20)$$

With the help of Eqs. (19) and (20) one can rewrite (16) and (17) as

$$\dot{\phi}^2 = \frac{\alpha^2}{3M_{\text{p}}^2 \Omega_{\Lambda}} \left[1 - \frac{1}{2 - \Omega_{\Lambda}} \left(1 - \frac{\Omega_k}{3} + 2b^2 \right) \right], \quad (21)$$

$$V(\phi) = \frac{\alpha^2}{6M_{\text{p}}^2 \Omega_{\Lambda}} \left[1 + \frac{1}{2 - \Omega_{\Lambda}} \left(1 - \frac{\Omega_k}{3} + 2b^2 \right) \right]. \quad (22)$$

At the present time, if one takes $\Omega_\Lambda = 0.73$ and $\Omega_k = 0.01$ then from Eq. (21) one can obtain a phantom scalar field ($\dot{\phi}^2 < 0$) provided $b^2 > 0.14$.

From definition $\dot{\phi} = \phi' H$ one can rewrite Eq. (21) as

$$\phi' = \sqrt{3}M_p \left[\Omega_\Lambda \left(1 - \frac{1}{2 - \Omega_\Lambda} \left(1 - \frac{\Omega_k}{3} \right) + 2b^2 \right) \right]^{\frac{1}{2}}, \quad (23)$$

where $\phi' = d\phi/d \ln a$.

Using Eqs. (3) and (1), the evolutionary form of the Chaplygin gas scalar field can be obtained as

$$\phi(a) - \phi(a_0) = \sqrt{3}M_p \int_{a_0}^a \left[\frac{\Omega_\Lambda}{2 - \Omega_\Lambda} \left(1 - \Omega_\Lambda + \frac{\Omega_k}{3} - 2b^2 \right) \right]^{1/2} \frac{da}{a}, \quad (24)$$

where a_0 is the scale factor at the present time. At late times when $\Omega_\Lambda \rightarrow 1$ and $\Omega_k \rightarrow 0$, Eqs. (22) and (24) yield

$$V(\phi) = (1 + b^2)\alpha H, \quad (25)$$

$$\phi(a) - \phi(a_0) = i\sqrt{6}M_p \ln \left(\frac{a}{a_0} \right). \quad (26)$$

The evolution of the Chaplygin gas scalar field (24) and the variation of the Chaplygin gas scalar potential (22) are plotted in Figs. 1 and 2, respectively.

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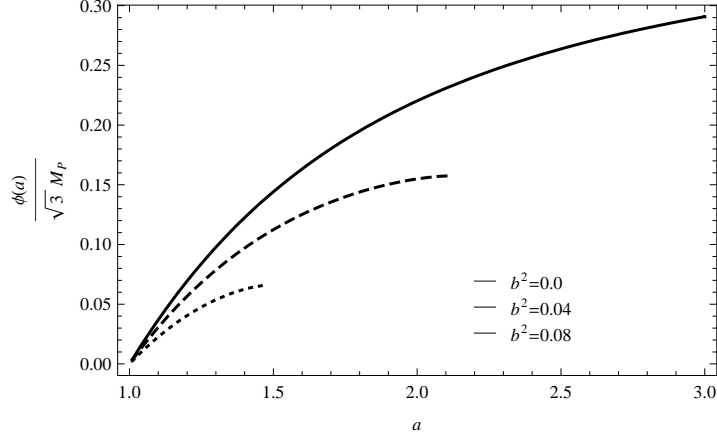


Figure 1: Chaplygin gas scalar field ϕ versus scale factor a for different values of the coupling constants $b^2 = 0$ (solid line), 0.04 (dashed line) and 0.08 (dotted line). Auxiliary parameters are: $\Omega_{\Lambda_0} = 0.73$, $\Omega_{k_0} = 0.01$ and $\phi(1) = 0.0$.

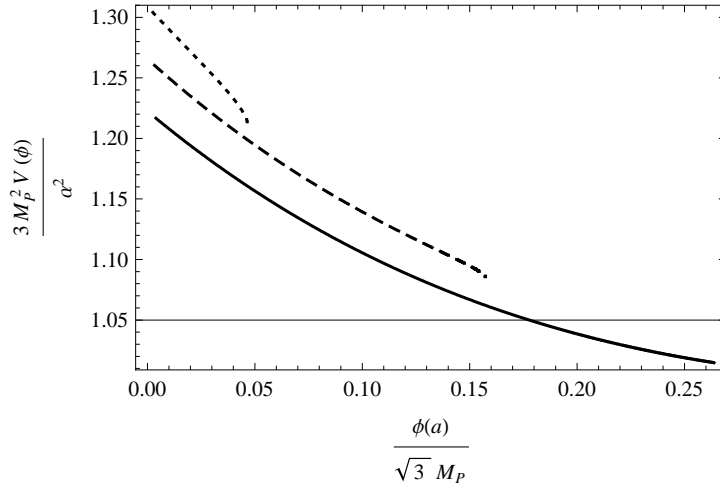


Figure 2: Chaplygin gas scalar potential $V(\phi)$ versus scalar field ϕ . Legend and auxiliary parameters as in Fig. 1.